Iterative Order Recursive Least Square Algorithm for Compressive Sensing-based Machine Type Communication (MTC)

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Abstract

This paper presents an Iterative Order Recursive Least Square based Orthogonal Matching Pursuit (IORLS-OMP) algorithm that can significantly reduce the complexity of decoding in a compressive sensing (CS) process. The algorithm is designed putting a CS-based delay-intolerant machine type communication (MTC) in mind where decoding complexity must be kept as low as possible. OMP algorithms constitutes of matrix inversions which makes up most part of their complexity. This complexity even gets worse when group of symbols are decoded together with Group Orthogonal Matching Pursuit (GOMP) for improving the accuracy. In this paper, it is shown that the new algorithm greatly reduces the computational complexity while increasing an accuracy of the GOMP algorithm by collecting node activity information from all the symbols transmitted in a frame.

1. Introduction

With the expected explosion of machine type communication (MTC) devices in the coming years, new medium access techniques that agree with the requirements and characteristics of them must be consider, as the existing systems, such as LTE, are inefficient and do not fulfill the expectations [1]. MTC is characterized by low rate sporadic communication, massive connectivity, small packet size, increased power efficiency and in some cases intolerance to latency. In this respect, various alternative solutions have been proposed. Exploiting the sporadic nature of MTC, i.e. devices transmit intermittently and hence not all the device would be active at a time, several CS based medium access techniques have been proposed [1][2].

CS is a novel paradigm which enables us to recover a sparse signal from a fewer number of samples than the Nyquist rate. Mathematically speaking, it is a method of obtaining a unique solution from an underdetermined linear system of equations under a sparse solution constraint. A discrete finite length signal \( x \in \mathbb{R}^n \) is said to be k-sparse if \( \| x \|_0 \leq k \) where \( \| . \|_0 \) is \( I_k \) pseudo-norm which counts the nonzero elements in the vector \( x \). Compact measurement of a sparse signal \( x \) is taken by a measurement matrix \( \Psi \in \mathbb{R}^{m \times n} \) as

\[
y = \Psi x \tag{1}
\]

When the system of equation in (1) is underdetermined, with the matrix \( \Psi \) being ‘fat’ (\( m < n \)), it is generally impossible to recover \( x \) from \( y \). But if a prior information about the sparsity of the vector \( x \) is injected to (1), \( x \) can be recovered by solving the convex minimization problem in (2) [2].

\[
x = \arg \min_{x \in \mathbb{R}^n} \| x \|_1 \quad \text{s.t.} \quad y = \Psi x \tag{2}
\]

Random matrices with independent and identically distributed entries are commonly used as measurement matrices. Such matrices can be drawn from Bernoulli, Gaussian or other distributions as long as their dimensions satisfy \( m \geq ck \log(n/k) \) for some constant \( c \). In CDMA like communication systems pseudorandom spreading sequences easily make up a measurement matrix in (1) and fulfill RIP.

As a low complex alternative of (2), greedy algorithms which iteratively build up the support of \( x \), has gained significant attention in the recent years. A simple and novel member of the greedy algorithms is orthogonal matching pursuit (OMP) which constructs support of \( x \) by iteratively selecting columns of \( \Psi \) which has the highest correlation with \( y \).

2. System Model

A star topology where \( N \) nodes communicate with a central aggregating base station is considered. Due to sporadic transmission assumptions only few of these nodes, \( K \) of them, transmit their packets at a frame time in a synchronized manner. An overloaded CDMA-like transmission is assumed where each node spread its symbols by a unique sequence, with length \( M \), assigned to it. Here the system is overloaded as \( M < N \).

Active node \( n \) transmit \( N_c \) symbols \( d_c = [d_{c,1}, d_{c,2}, \ldots, d_{c,N_c}] \) per each frame time drawn from an alphabet \( \mathbf{A} \) and inactive nodes are modeled to transmit an all-zero frame. The received signal by the base station over a symbol time is given by (3).

\[
y_i = \mathbf{A} c_i + \omega_i \text{ for } i = 0, 1, \ldots, N_c - 1 \tag{3}
\]

\( \omega_i \in \mathbb{R}^M \) is additive noise at the \( i^{th} \) symbol and the matrix \( \mathbf{A} \in \mathbb{R}^{M \times N_c} \) would hold the effect of the spreading and the channel altogether. Furthermore, when group of symbols are decoded by the GOMP algorithm the received signal model in (3) is modified as

\[
\bar{y}_v = \bar{A} \mathbf{x}_v + \bar{\omega}_v \text{ for } v = 1, 2, \ldots, N_c / L \tag{4}
\]

where \( \bar{\mathbf{x}}_v = [d_{c,1}, d_{c,2}, \ldots, d_{c,L(v-1)+1}, d_{c,L(v)-1}, \ldots, d_{c,L}] \) is a vector constituting \( L \) symbols from each of \( N \) devices, \( \bar{\mathbf{X}}_v \in \mathbb{R}^{LM \times LN} \) is build by shifting and interleaving the columns of \( \mathbf{A} \) in (3) so that they would spread \( L \) symbols from each user.

3. IORLS-OMP algorithm

On each iteration of the IORLS-OMP algorithm, which iteratively employs the OMP algorithm, the weight \( \mathbf{W} \) to select the support of \( x \) is updated by

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Fig. 1 shows the IORLS-OMP algorithm in the pseudocode below. The outer iteration at lines 2-13 is performed in parallel over all symbol times in a frame. The $K$ most likely transmitted spreading sequences, i.e., columns of $A$, are selected (line 7) by the inner iteration at lines 5-11. As now we have information about the most probable $K$ active sequences from each of $N_c$ symbols, new weights can be computed favoring those sequences found to be active in most of the symbol times. These weights, which were initially one, will in turn be used for the next iteration. Here, one should notice the inner iteration is simply the OMP algorithm with slight modifications. At the $l$th iteration, the column of $A$ with maximum correlation with the residual $r^{l-1}$ is selected (line 7), the index of this column is added to the support set $\Gamma(G)$ (line 8), we estimate the transmitted symbols based on the detected supports (line 9), and the residual is updated to orthogonal projection of received signal on the already estimated symbols. One should also note here, to estimate the symbols at line 9 ordered least square (OLS) approach is employed which does not constitute matrix inversions.

\[
W_m = \left| \frac{d}{d} \right| / N_c
\]  

(5)

The computational complexity of the OMP algorithm in terms of floating point operations (FLOPs) is given in [3]. Taking a similar approach we can drive the FLOPs required for GOMP and IORLS as summarized in Table 1 in which $l$ denotes number of iterations.

\[
\Gamma(l) = \Gamma(l-1) \cup k_{\max}
\]  

(6)

\[
D_{\Gamma(l)} = \left[ \begin{array}{c} D_{\Gamma(l-1)} \nonumber \n A_{l}^{\dagger} A_{l} & A_{l}^{\dagger} P_{l}^{s} A_{l} \nonumber \n A_{l}^{\dagger} P_{l}^{s} A_{l} & A_{l}^{\dagger} P_{l}^{s} A_{l} \nonumber \end{array} \right]
\]  

(7)

and

\[
P_{l}^{s} = I - G_{l}^{D_{l}} G_{l}^{T}
\]  

(8)

The outer iteration at line 2-13 is performed in parallel over all symbol times in a frame. The $K$ most likely transmitted spreading sequences, i.e., columns of $A$, are selected (line 7) by the inner iteration at lines 5-11. As now we have information about the most probable $K$ active sequences from each of $N_c$ symbols, new weights can be computed favoring those sequences found to be active in most of the symbol times. These weights, which were initially one, will in turn be used for the next iteration. Here, one should notice the inner iteration is simply the OMP algorithm with slight modifications. At the $l$th iteration, the column of $A$ with maximum correlation with the residual $r^{l-1}$ is selected (line 7), the index of this column is added to the support set $\Gamma(G)$ (line 8), we estimate the transmitted symbols based on the detected supports (line 9), and the residual is updated to orthogonal projection of received signal on the already estimated symbols. One should also note here, to estimate the symbols at line 9 ordered least square (OLS) approach is employed which does not constitute matrix inversions.

1: iteration = 0, $W = I$
2: repeat
3: iteration ← iteration + 1
4: $G^{l} = \phi, l = 0, r^{0} = y$,
5: repeat
6: $l ← l + 1$
7: $k_{\max} = \arg \max_{k} \sum_{I=1}^{I} | A_{l,k}^{\dagger} w^{l-1} |$ with $k \in \Gamma(G^{l-1})$
8: $\Gamma(G) = \Gamma(G^{l-1}) \cup k_{\max}$
9: solve $t_{\Gamma}^{l-1}$ using (6) and $y_{\Gamma}^{l-1} = 0$
10: $r^{l} = y_{\Gamma}^{l-1} - A x_{\Gamma}^{l-1}$
11: until $l = K$, or $| r^{l} | < | r^{0} |$
12: update $W$ using (5)
13: until max _iteration

Figure 1. IORLS-OMP algorithm

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>FLOPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMP</td>
<td>$3MK^2 + 2MNK + O(KM)$</td>
</tr>
<tr>
<td>GOMP</td>
<td>$3MK^2 + 2MNK + O(KM)$</td>
</tr>
<tr>
<td>IORLS-OMP</td>
<td>$I(3MK^2 + 4MNK) + O(KM)$</td>
</tr>
</tbody>
</table>

Table 1. Time complexity of the different algorithms

4. Simulation Results

To evaluate the modified algorithm a simulation is setup by using the parameters in Table 2. Fig. 2(a) depicts the FLOPs saving of the new algorithm compared to GOMP algorithm. This saving gets more substantial as the sparsity increases. Fig. 2(b) shows that the new algorithm greatly enhances the probability for perfect recovery by just applying few iterations ($I = 3$) in the simulation. This gain comes from the fact that the new algorithm utilizes the node activity information gathered from all symbols in the frame. As a final remark, this advantage can be exploited from any problem where there is frame-wise sparsity.

5. Conclusion

In this paper, we proposed an IORLS-OMP algorithm which can substantially reduce the decoding time of GOMP algorithm for CS-based machine type communication. The proposed algorithm has also shown to have a higher probability of perfect recovery. In CS problems, where there is frame-wise sparsity, the activity information of a node can be gathered from all symbols in a frame, which is the reason behind the increase in recovery probability.

References