Performance analysis of ZFBF in spatially correlated massive MIMO environments
Yong-Suk Byun, Hyung-Keon Kim and Yong-Hwan Lee
School of Electrical Engineering and INMC
Seoul National University
E-mail: {ysbyun, kgheee}@ttl.snu.ac.kr and ylee@snu.ac.kr

Abstract
Massive MIMO systems may experience channel correlation between the transmit antennas, referred to spatial correlation, significantly affecting transmission performance. In this paper, we analyze the performance of zero-forcing beamforming with the use of channel information with some delay in spatially correlated massive MIMO channel environments when the channel information is available with some delay. We verify the analytic results by computer simulation.

I. Introduction
Mobile data traffic exponentially grows according to the activation of wireless multimedia services associated with prevalence of smart phones, social networking services, and device-to-device communications [1]. As a consequence, wireless communication systems should be able to handle so-called big data environments. It is well known that the bandwidth efficiency (or the spectral efficiency) can be improved as the number of transmit antennas increases [2]. However, the theoretical performance may not be achievable in practice due to implementation issues including the presence of channel correlation.

A number of research works have analyzed the capacity of transmit beamforming schemes [3-5]. However, they did not consider the spatial channel correlation in the analysis. The spatial channel correlation between the transmit antennas may cause channel directivity, affecting the system capacity. In this paper, we analyze the expected SINR of zero-forcing beamforming (ZFBF) in spatially correlated massive MIMO (m-MIMO) channel environments.

The remainder of this paper is organized as follows. The system model considered is described in Section II. The capacity of ZFBF beamforming scheme is analyzed in the presence of spatial channel correlation in Section III. The performance is evaluated by computer simulation in Section IV. Finally, conclusions are given in Section V.

II. System model
We consider the downlink of a multi-user wireless communication system with (m-MIMO) schemes. We assume that the base station (BS) simultaneously transmits signals to $K$ users using $N_T$ antennas. When the signal is transmitted by means of multi-antenna techniques, the received signal of user $k$ with a single receive antenna can be represented as

$$y_k = \sqrt{\alpha_k P_k} h_k w_k s_k + \sum_{i \neq k} \sqrt{\alpha_i P_i} h_i w_i s_i + n_k$$ (1)

where $\alpha_k$ denotes the large-scale fading coefficient from the BS to user $k$, $h_k$ denotes the $(1 \times N_T)$ channel vector of user $k$, $w_k$ denotes the $(N_T \times 1)$ normalized beam weight vector for user $k$, $P_k$ denotes the transmission power for user $k$, $s_k$ is the data symbol transmitted to user $k$ with $E[|s_k|^2] = 1$, and $n_k$ denotes additive zero-mean white Gaussian noise (AWGN) with variance $\sigma_n^2$.

When the channel experiences spatially correlated Rayleigh fading, it can be represented as,

$$h_k = h_k R_{kk}^{\frac{1}{2}}$$ (2)

where $h_k$ denotes an uncorrelated channel vector of user $k$ whose elements are independent and identically distributed (i.i.d) zero-mean complex Gaussian random variables with unit variance, and $R_{kk}$ denotes the $(N_T \times N_T)$ transmit spatial correlation matrix. Since $R_{kk}$ is a positive semidefinite Hermitian matrix, it can be represented as:

$$R_{kk} = U_{kk} \Lambda_{kk} U_{kk}^H$$ (3)

where $U_{kk} = [u_{k1}, u_{k2}, \ldots, u_{kN_T}]$ is an $(N_T \times N_T)$ unitary matrix whose columns are the normalized eigenvectors of $R_{kk}$ and $\Lambda_{kk}$ is a diagonal matrix whose diagonal elements are the eigenvalues of $R_{kk}$, represented as $\{\lambda_{k1}, \lambda_{k2}, \ldots, \lambda_{kN_T}\}$, where $\lambda_{k1} \geq \lambda_{k2} \geq \ldots \geq \lambda_{kN_T} \geq 0$. We define $u_{k\max}$ by the principal eigenvector corresponding to the largest eigenvalue $\lambda_{k1}$ of $R_{kk}$ (i.e., $u_{k\max} = u_{k1}$).

III. Performance analysis
When the BS transmit signal by means of ZFBF, the corresponding precoding matrix can be determined as

$$F = H H^H = f_1 f_1^H \cdots f_K f_K^H$$ (4)

where $H$ denotes the $(K \times N_T)$ channel matrix whose column vectors are $h_k^H$ and $F$ denotes the $(N_T \times K)$ precoding matrix comprising $K$ column vectors $\{f_k\}$. The normalized transmit beamforming weight of user $k$ can be determined as $w_k = f_k / \|f_k\|$, where $\|F\|$ denotes the norm of the vector $F$.

Since the ZFBF can perfectly remove the multi-user interference term (i.e., $\|h_l f_k\|^2 = 0$ for $l \neq k$) and $\|h_k f_k\|^2 = 1$, the SINR of user $k$ with the use of ZFBF can be represented as

$$\gamma_k \approx \frac{\alpha_k P_k}{\sigma_n^2} \frac{1}{\|f_k\|^2}$$ (5)

The array gain of ZFBF can be represented as [3].
\[
\frac{1}{\|f_k\|^2} = \frac{\det[H^H]}{\det[H^{(k)}H^{-1}(k)H^{H}]} \quad (6)
\]

where \(H^{(i)}\) agrees with the channel matrix \(H\) except the \(k\)-th row corresponding to the channel from the BS to user \(k\), and \(\det[.]\) denotes the determinant of a matrix. Without loss of generality, we will consider \(k = 1\) in what follows. Since [3]

\[
\frac{1}{\|f_1\|^2} = \sum_{m=1}^{\infty} (-1)^m h_m h_m^H \frac{\det[H^{(1)}H^{H}]}{\det[H^{(1)}H^{-1}(1)H^{(1)H}]} \quad (7)
\]

it can be seen that

\[
E\left\{\left(\sum_{j=2}^{\infty} (-1)^j h_j h_j^H \frac{\det[M_j]}{\det[H^{(1)}H^{-1}(1)H^{(1)H}]}\right)\right\} = E\left\{\frac{\det[M_1]}{\det[H^{(1)}H^{-1}(1)H^{(1)H}]} h_1 h_1^H \right\} = \frac{E\{h_1 A h_1^H\}}{N_t} \quad (8)
\]

where \(M_j\) denotes the sub-block matrix of \(H^{(j)}H^{-1}(j)H^{(j)H}\) after removing the \(i\)-th row and \(m\)-th column. Since \(h_1\), \(h_2\), and \(h_m\) are independent each other, it can be shown from the law of large numbers that

\[
\frac{\sum_{j=2}^{\infty} (-1)^j \det[M_j]}{\sum_{j=2}^{\infty} (-1)^j \det[M_j]} = \frac{E\{h_1 A h_1^H\}}{N_t} \quad (9)
\]

It can be shown from \(E\{h_1 A h_1^H\} = tr(A)\) that

\[
E\{h_k A h_k^H\} = E\{h_k R_k^{(2)} R_k h_k^H\} = tr(U_k^H A U_k) \quad (10)
\]

where \(U_k\) is a diagonal matrix whose \(i\)-th diagonal elements are \(\lambda_{k,m} = \lambda_{k,m,n} \lambda_{k,n,m}^{-1} \lambda_{n,m}^{-1} u_k u_k^H\). Thus, it can be seen that

\[
E\{h_k R_k h_k^H\} = \sum_{j=1}^{\infty} \lambda_{k,m} \lambda_{k,n} \lambda_{n,m}^{-1} |u_k|^2 |u_n|^2 \quad (11)
\]

The array gain of ZFBF, \(\beta_k\), can be represented as

\[
\beta_k = \frac{1}{E\{\|f_k\|^2\}} = E\{\|h_k^H\|^2\} = \frac{\sum_{j=1}^{\infty} \lambda_{k,m,n}^{-1} N_t}{N_t} \quad (12)
\]

Finally, the average SINR of user \(k\) with the use of ZFBF can be represented as

\[
\overline{\gamma}_k \approx \frac{\alpha_k P_k}{\sigma_k^2} \left(1 - \frac{\sum_{m=1}^{\infty} \lambda_{m,n}^{-1} N_t}{N_t}\right) \quad (13)
\]

IV. Performance evaluation

The analytic results are verified by computer simulation. We assume that the BS transmit signal using transmit antennas of up to 64 by means of ZFBF.

Fig. 1 depicts the effective SINR of ZFBF for uniformly distributed users according to the number of transmit antennas when the spatial correlation is 0.3 and 0.9, and the spatial multiplexing order is 4, 8, and 16. It can be seen that as the spatial correlation decreases, the SINR of ZFBF is increases. In high spatially correlated channel environments, the channel has a smaller degree of freedom, which may reduce the array gain.

However, the performance of ZFBF is little affected by the channel correlation provided that the number of antennas is much larger than the multiplexing order. This is mainly because the degree of freedom is large enough to ignore the effect of channel correlation. It can also be seen that as multiplexing order increases, the average SINR decreases mainly due to the decrease of transmit power for each beam and the array gain of ZFBF. It can also be seen that the SINR analysis quite agrees well with the simulation results when the number of antennas are larger than the multiplexing order.

V. Conclusion

In this paper, we have analyzed the performance of ZFBF in terms of the SINR in spatially correlated massive MIMO channel environments. It has been shown that the performance of ZFBF is strongly affected by the channel correlation with the use of a small number of antennas, but not much with the use of antennas much larger than the multiplexing order. The simulation results show that the performance analysis is quite accurate when the number of antennas are larger than the multiplexing order.

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Figure 1. Average SINR of ZFBF according to number of antennas

Reference